THE *General Radio* EXPERIMENTER

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ELECTRICAL MEASUREMENTS AND THEIR INDUSTRIAL APPLICATIONS

Also
IN THIS ISSUE

THE COMMUNICATIONS RECEIVER AS A WAVE ANALYZER

● A GOOD MANY laboratory measurements would be considerably simplified if there were available a device to do at radio frequencies the job that the familiar heterodyne type of wave analyzer does at audio frequencies.

Nearly every laboratory, however, has a communications type of radio receiver which can be quite easily adapted to the problem. While the input impedance may not be so high, and the output meter and the frequency dial calibration may not be so accurate as may be desired in many cases, such inadequacies can often be remedied by proper use and calibration. Standard-signal generators and frequency meters are readily used for this calibration.

The simplicity with which an elementary type of analysis can be made with a communications receiver is illustrated by its use for determining the length of short, periodically-repeated pulses of voltage. Harmonic generators[1] and Class C amplifiers often operate with such pulses of plate current. The adjustment for proper operation in these cases is usually well indicated by setting for the desired length of current flow, and a simple technique for the measurement of this length provides an additional tool in this adjustment.

[1]F. R. Stansel, "Some Analyses of Wave Shapes Used in Harmonic Producers," *Bell System Technical Journal*, v. 20, n. 3, July, 1941, pp. 331-339.

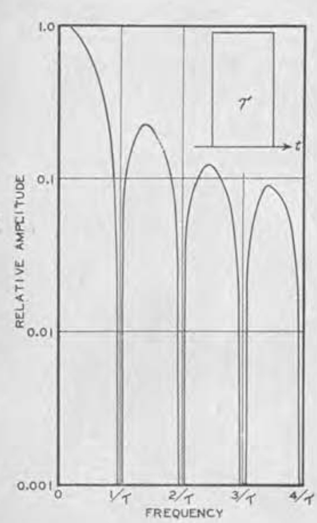


FIGURE 1. Envelope of the frequency spectrum of a rectangular pulse.

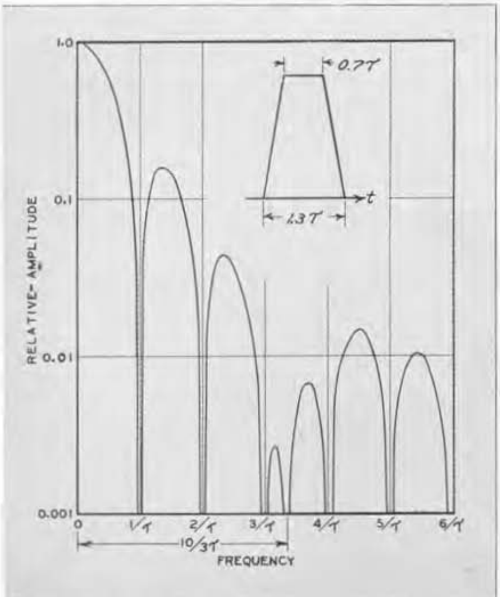
The waveform that is generally obtained in a system of this type can be well represented by a short pulse that is a section of a sine wave. The angle of current flow, θ, is the usual measure of the length of such a pulse and is related to its length in seconds, τ, by the expression

$$\theta = 2\pi f \tau$$

where f is the frequency of the fundamental driving voltage.

In many cases the pulse is so flat on the top that it is better represented by a trapezoid with sides of finite slope. As a limiting case it can be considered rectangular, and, because the analysis and measurement for the rectangular shape is simple, it will be considered first.

The length of a rectangular pulse can be determined from the locations of the minima of intensity of the components in its frequency spectrum. For a truly rectangular pulse, the minima (zeros of the envelope) occur at integral multiples of the frequency whose period is equal to the length of the pulse. Thus a 10-microsecond pulse has a spectrum with minima of intensity at 100 kc, 200 kc, 300 kc, etc.; and a 1-micro second pulse has minima at 1 Mc, 2 Mc, 3 Mc, etc.



The Fourier analysis of a rectangular pulse[2] yields a frequency spectrum of the form shown in Figure 1. Here only the envelope of the amplitude of the components is plotted. A pulse that is periodically repeated has a line spectrum with discrete components at integral multiples of the frequency of repetition, and the actual minima will in general not occur at exactly the above values. Thus, if the repetition rate is 1100 cps for the 10-microsecond pulse, components can be found at 1100, 2200, 3300, etc., cps. The amplitude of these components will decrease gradually with increasing frequency until the first minimum at 100,100 cps, beyond which the amplitude increases to a secondary maximum, decreasing again to the second minimum at 200,200 cps. Succeeding minima occur at 300,300; 400,400; 500,500; 599,500; 699,600; etc., cps.

If a rectangular pulse of voltage is impressed on a communications receiver, the general nature of the spectrum can readily be observed on the output meter as the receiver is tuned over its range. Then the frequencies at which minima occur can be readily located. The simple relationship between the location of the minima of the frequency spectrum and the length of the pulse then makes possible the calculation of the length of the pulse.

Because of the relatively broad band of reception (except with the use of quartz-crystal filters) at audio-frequency repetition rates, the response may appear to follow the envelope of the spectrum instead of giving the pattern of discrete responses that one might at first

[2]The rectangular pulse is analyzed in many texts. The article by Stansel (see Footnote 1) considers all the wave shapes treated here.

FIGURE 2. Envelope of the frequency spectrum of a trapezoidal pulse. It is very similar to the envelope of Figure 1 but has a double set of zeros.

expect. This envelope response is generally advantageous for locating the minima, in that a smooth drop to a minimum as the receiver tuning is varied makes possible rapid location of each one. In addition, since the sensitivity of the receiver is not perfectly uniform over its range, the frequency spectrum may appear to be somewhat different from the expected, but usually the distortion is not troublesome.

One precaution worth observing is that the receiver should not be overloaded. If overloading occurs, the frequency spectrum may be altered sufficiently to make the measurements useless. However, except for very low repetition rates and impulses of extremely short duration, one can usually obtain adequate measurements of the frequencies of the minima with relatively low inputs. Extraneous signals may cause spurious responses to be obtained, and the apparatus therefore should be arranged to avoid pickup of unwanted signals.

Although the foregoing has been based on a pulse of truly rectangular shape, the procedure can also be used for pulses that are only approximately rectangular. In actual practice the sides of a pulse have a finite slope, and a trapezoidal pulse (actually of an isosceles-trapezoid shape) is a better approximation to the usual case.

The envelope of the frequency spectrum of a pulse whose base is 86% longer than its top is shown in Figure 2. Two sets of minima occur. Those in the first set are integral multiples of the fundamental frequency whose period is the average length of the pulse. This relationship is true in general for trapezoidal pulses, and it is normally the only one of interest.

The minima in the second set occur at integral multiples of the frequency whose period is one-half the difference in length of the base and the top. This latter difference is often small, with the result that only a few such minima occur for many minima of the set corresponding to the average length of the pulse. For the extreme case of the triangular wave shape the two sets are identical, and the average length is the only one obtained.

The pulse that is a section of a sinusoid has a series of zeros that are not so simply related to the length of the pulse. If the section is a complete half wave, the zeros occur at frequencies that are 3/2, 5/2, 7/2, etc., multiples of the frequency whose period is equal to the length of the pulse. This case is the familiar one of the half-wave rectifier whose output has no components of the

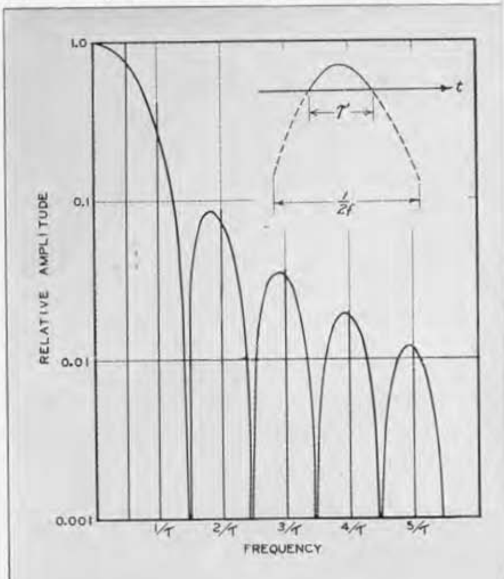


Figure 3. Envelope of the frequency spectrum of a pulse that is a very small section of a sine wave. When the pulse is a half sine wave repeated at the sine-wave fundamental frequency, the relative amplitudes of the components of the frequency spectrum are as shown by the vertical lines in the plot. The amplitudes for intermediate pulse lengths will lie between the limiting values shown here.

3rd, 5th, 7th, etc., harmonic of the fundamental frequency.

As the pulse becomes a smaller section of a sine wave, these minima shift downward in frequency, reaching in the limit 1.430, 2.459, 3.471, 4.477, 5.482, 6.484, etc., times the frequency whose period is equal to the length of the pulse. The shift from the simpler values given above is small, becoming smaller with each succeeding minimum. The envelope for a very short pulse is shown in Figure 3.

It is interesting to notice that the average length, considered as the area divided by the maximum height, for a half sinusoid, is $2\tau/\pi$ and that for a section of a sine wave which represents only a small angle of flow is $2\tau/3$. Thus, the first minimum is approximately at the frequency whose period is equal to the average length of the pulse. However, each succeeding minimum is spaced from the one preceding it by approximately the frequency whose period is equal to the total length, τ.

— ARNOLD PETERSON

CONTINUOUS INTERPOLATION METHODS

● **THE FREQUENCY CALIBRATION METHODS** which have been outlined previously[1] are equivalent to point-by-point interpolation, where the frequency of an oscillator could be set to, or measured against, the nearest multiple of a low audio-frequency standard. We now consider methods for setting an oscillator to, or measuring the frequency of an oscillator at, any point in the frequency range.

The basis of continuous interpolation is quite generally a continuously variable oscillator, usually in the audio-frequency or low radio-frequency range. Such an oscillator is designed to be as stable as possible and to permit calibration on a greatly expanded scale. In general, the over-all accuracy of measurement depends upon the absolute accuracy (in cycles per second) of the interpolation oscillator, and for greatest absolute accuracy this requires that the oscillator be in the low-frequency ranges.

The problem is indicated in the very simple diagram of Figure 1. The frequency to be measured, or the desired final value of frequency, is shown at f_x, lying anywhere between two known multiples of a standard frequency f_s, nf_s, and $(n + 1) f_s$. Since all interpolation systems are based on the interpretation of this elementary diagram, it is worth considering in greater detail.

Firstly, *assuming* nf_s *and* $(n + 1) f_s$ *to exist as physical frequencies*, that is, that voltages of these frequencies exist in usable magnitude, f_x can be evaluated in several ways, as follows:

(1) An auxiliary radio-frequency oscillator is used, generally called a "Heterodyne Frequency Meter." The dial settings of this oscillator are determined for nf_s, f_x, and $(n + 1) f_s$ in turn;

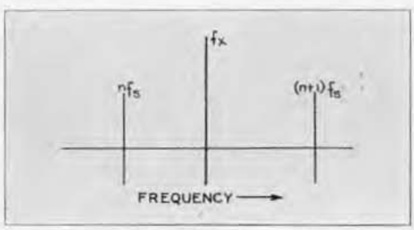


FIGURE 1. Basic Interpolation Diagram.

[1] J. K. Clapp, "Calibration of Equipment in the Low and Medium Radio-Frequency Ranges, in Small Steps of Frequency," General Radio *Experimenter*, Vol. XVIII, No. 5, October, 1943.

f_x is then found by linear interpolation, as will be shown later.

(2) The frequencies f_x and nf_s, or f_x and $(n + 1) f_s$, are impressed on a detector (or receiver) and the *lower* beat frequency difference produced in the detector is obtained. This beat frequency ranges from 0 to $f_s/2$ cycles. The magnitude of the frequency difference is measured, and its sign determined, by methods considered below.

(3) The frequencies f_x and nf_s, or f_x and $(n + 1) f_s$, are impressed on a detector, and the *higher* beat frequency difference produced in the detector is obtained. This beat frequency ranges from $f_s/2$ to f_s cycles. The magnitude of this difference is measured, and its sign determined, by methods described later.

(4) If the standard frequency can be *raised*, by a smoothly acting fine frequency control, the harmonic nf_s can be slid along the frequency scale until it is brought exactly to the value f_x. The amount that nf_s is raised to accomplish this can be determined from the frequency control settings; the sign of the change is always positive.

Secondly, *assuming that nf_s and $(n + 1) f_s$ do not exist physically*, f_x can still be evaluated in terms of these multiples of the standard frequency f_s, as follows:

(5) The *difference* between f_x and the known frequency of an interpolation oscillator, f_i, is made equal to a multiple of the standard frequency f_s, using cathode-ray oscillograph patterns. The important point to notice is that this method permits measurements to be made at high multiples of a standard frequency, where the standard source has *no harmonics*. The multiples exist only as patterns on the oscilloscope screen.

Methods (1) and (2) are considered in the following paragraphs, where factors affecting the operation are detailed. Methods (3) and (4) will be discussed in a forthcoming issue of the *Experimenter*.

The methods are summarized in the accompanying diagrams, where the essential frequency relationships are illustrated at the top of the figure, followed by a block diagram below. The equations pertaining to each method are followed by a brief listing of the principal limitations and advantages.

Method 1: Direct Interpolation

This method would not be used where precise results are required, principally because of the basic fault that interpolation is accomplished by using only a very small portion of the working scale. To obtain fine interpolation, then, it is necessary to "magnify" this small portion of the scale which, in this case, can only be accomplished by mechanical means. These are subject to severe limitations, as the list indicates.

The method is still of value for rough work, or where measurements of moderate accuracy need to be made only occasionally.

Method 2: Direct Beating

This method is widely used and is capable of high precision. It has certain limitations, particularly as to the measurement of very high frequencies. Because of its general utility, however, the details below may be helpful.

If the frequency f_x is that of a remote transmitter, or one of several local transmitters or oscillators, radio frequency selectivity, as provided by the receiver in the diagram, is a practical necessity.

Where interference or noise conditions permit, the nuisance of having to tune the receiver to each value of f_x can be avoided by using an untuned detector for the receiver. Generally this results in some decrease in the upper limit of range for f_x because of the decreased sensitivity.

A compromise arrangement, which is useful if the unknown frequency is intermittent (as, for instance, that of a keyed transmitter), is to use an untuned detector with a local substitution oscillator. This oscillator is set to zero beat, or matched, with the unknown frequency. The fundamental frequency of the oscillator is then measured against the standard.

By using harmonics of the substitution oscillator to match the unknown frequency, the effective upper limit of measurement can be greatly increased. By choosing particular harmonics, which can be done easily whenever an approximate value for f_x is known, simple or decimal multipliers can be maintained for all measurements.

For example, when 10 kc standard frequency harmonics are used it is difficult to make measurements above 8 to 10 Mc. When a substitution oscillator is used, if the 10th harmonic is used for matching to f_x, then the final result is always 10 times the measured oscillator fundamental frequency, requiring no computation.

In common with other methods of measuring frequencies, some difficulty is encountered in measuring frequencies which are only a few cycles removed from a standard harmonic frequency, or which are very nearly halfway between two standard harmonic frequencies. The first can be avoided by substituting a new value of standard frequency through the use of a 9 and 11 kc multivibrator, instead of 10 kc, which effectively transfers the small frequency difference to appear as that same difference from a multiple of 1 kc, where it is easily measured. This holds for all frequencies except multiples of 990 kc ($9 \times 10 \times 11$). A similar substitution can be made by using an off-set standard[2]. A third possibility is to use an oscillating receiver, tuned so that a beat tone of about 1 kc is obtained against the standard harmonic frequency. The beat tone obtained against the unknown frequency will be only a few cycles different. The combination acts as a tone of about 1 kc, waxing and waning at a rate equal to the difference of the unknown and standard frequencies. The "flutter" can be matched with an interpolation oscillator using a cathode-ray oscilloscope[3].

When the frequency to be measured lies nearly halfway between two standard frequency harmonics, the receiver selectivity can be used to favor the beat obtained with one of the two standard frequencies, at the expense of the other. In cases where the selectivity is not adequate, or where an untuned detector is used, the standard frequency can be doubled. In effect, this eliminates all odd-numbered harmonics from the original harmonic series, and only a single beat frequency difference remains to be measured.

In attempting to extend the measuring range toward higher frequencies, several problems arise. First, with a given standard frequency, say 10 kc, measurements at higher frequencies require correspondingly higher multiples

[2] J. K. Clapp, "Identification of Harmonics in a Harmonic Series," General Radio *Experimenter*, Vol. XVIII, No. 4, September, 1943.

[3] J. K. Clapp, "Using the Cathode-Ray Oscillograph in Frequency Comparisons," General Radio *Experimenter*, Vol. XVI, No. 7, December, 1941.

of the standard frequency. It is very difficult to get harmonics of usable amplitude much beyond the 1000th harmonic. Some improvement can be had by modulating a higher frequency multivibrator with the output of a lower frequency one. For example, substantially stronger 10 kc harmonics can be obtained by using a 50 kc multivibrator modulated by a 10 kc multivibrator. A control for the amount of modulation must be provided since the amplitudes of different multiples of 10 kc vary substantially with different amounts of modulation.

An accompanying difficulty in this case centers on identifying the standard frequencies. This becomes more and more difficult as the number of used harmonics is increased. Where the modulated multivibrator system is used, the modulation can be reduced to zero and the nearest harmonic of the high frequency multivibrator can be identified. On reintroducing the modulation, the number of low-frequency harmonics passed over to reach the used harmonic can be counted.

If a higher standard frequency is used, to avoid the use of high multiples, other problems arise. Firstly, the pass-band of the receiver must be made wider, the higher the standard frequency. Secondly, the interpolating equipment must have a corresponding increase in range, usually accompanied by a corresponding loss in absolute accuracy. Since the interpolating equipment must be capable of measuring from 0 to $f_s/2$ cycles, any substantial increase in f_s means a great extension in the range of the interpolating equipment.

SUMMARY

Method 1 : Direct Interpolation

Using an oscillator (Heterodyne Frequency Meter), and assuming the calibration to be straight-line-frequency over the used portion, set for zero beat successively with the three frequencies nf_s, f_x, and $(n+1)f_s$, calling the respective dial readings θ_n, θ_x, θ_{n+1}. Then, from the figure

$$\frac{f_x - nf_s}{(n+1)f_s - nf_s} = \frac{\theta_x - \theta_n}{\theta_{n+1} - \theta_n} = \frac{f_x - nf_s}{f_s}$$

From which,

$$f_x = nf_s + \frac{\theta_x - \theta_n}{\theta_{n+1} - \theta_n} f_s$$

when the measurement is referred to the lower standard frequency harmonic, nf_s. Also,

$$\frac{(n+1)f_s - f_x}{(n+1)f_s - nf_s} = \frac{\theta_{n+1} - \theta_x}{\theta_{n+1} - \theta_n} = \frac{(n+1)f_s - f_x}{f_s}$$

From which

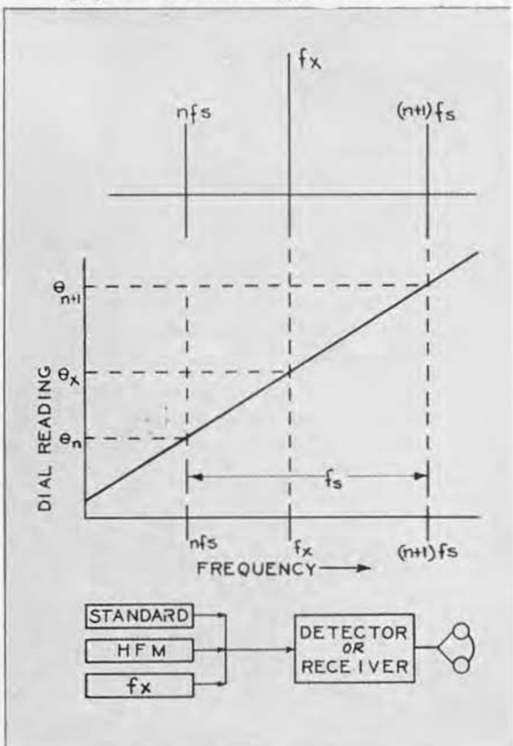
$$f_x = (n+1)f_s - \frac{\theta_{n+1} - \theta_x}{\theta_{n+1} - \theta_n} f_s$$

when the measurement is referred to the higher standard frequency harmonic, $(n+1)f_s$.

Principal Limitations of this method are:

(*a*) Linearity of calibration curve of Heterodyne Frequency Meter.

FIGURE 2. Diagrams illustrating method and equipment for direct interpolation.



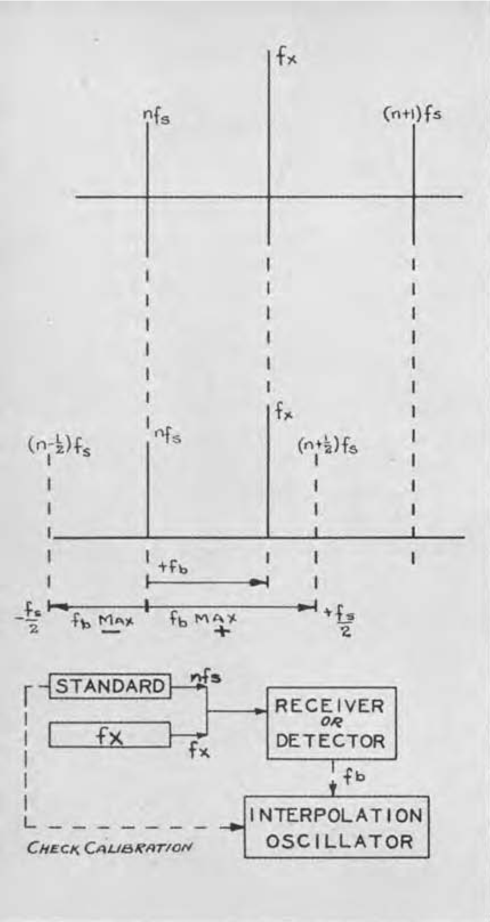


FIGURE 3. Diagrams illustrating method and equipment for direct beating.

(b) Fineness of graduation of the scale.
(c) Mechanical errors of drive, such as eccentricity, backlash, worm corrections, etc.
(d) Accuracy of setting for zero beat.
(e) Accuracy of calculations, or of reading charts, for solving equation.
(f) Use of only a very small portion of the scale for making interpolations.

Principal Advantages are:
(a) Simplicity of equipment.
(b) Simplicity of operation.
(c) Values of nf_s and $(n+1)f_s$ are known at once from heterodyne frequency meter calibration.

Method 2: Direct Beating

The standard frequency nf_s and the unknown frequency f_x are impressed on the receiver, which is tuned to accept both. The lower beat frequency difference $+f_b = f_x - nf_s$ (or $-f_b = nf_s - f_x$) is obtained in the receiver output and is measured by matching with the interpolation oscillator, covering a range from 0 to $f_s/2$ cycles. Then $f_x = nf_s \pm f_b$.

Principal Limitations of this method are:
(a) Standard frequency harmonics must be of usable magnitude in region around f_x.
(b) The frequency of the used standard frequency harmonic, nf_s, must be determined.
(c) The sign of the beat frequency, f_b, must be determined.
(d) The over-all accuracy depends on the accuracy of the interpolation oscillator and how carefully it is matched to f_b.

Principal Advantages are:
(a) Relatively high absolute accuracy is possible, since the entire range of the interpolating equipment is devoted to measuring f_b which is only a small part of the final result.
(b) The absolute accuracy is the accuracy *in cycles* with which f_b can be measured and is constant over a very wide frequency range.

—J. K. CLAPP

(To be continued)

ERRATA

Readers who were mystified by the relation between Equations (11) and (10) on page 1 of the December *Experimenter* will be relieved to learn that the exponent (½) was omitted from Equation (10), which should read

$$R_{in} \approx R_0\left[1 - \left(\frac{X_0 + X_T}{R_{in}}\right)^2\right]^{1/2} \quad (10)$$

In addition, a factor of ½ was omitted in the numerical expression for R_0 on page 2 which should read

$$R_0 = 71.2\left[1 + \frac{1}{2}\left(\frac{2.1 + 10.3}{71.2}\right)^2\right]$$

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